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Four-phase sphere modeling of effective bulk modulus of concrete

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Abstract

A four-phase sphere model, extended from Christensen and Lo's three-phase sphere model for two-phase composite materials, was proposed to estimate the effective bulk modulus of three-phase concrete. The formulations were developed by reducing the four-phase sphere model to an equivalent three-phase sphere model and an equivalent two-phase sphere model. A distinctive characteristic of the proposed model is that, in addition to considering other physical-mechanical parameters, it is able to evaluate the effect of the maximum aggregate size and aggregate gradation on the effective bulk modulus of concrete. Reasonable agreement was found between the calculated effective Young's modulus and the experimental results from the literature. This suggests that the proposed four-phase sphere model is suitable for estimating the effective elastic modulus of concrete. It is found that the maximum aggregate size, aggregate gradation, and the interfacial transition zone have a significant effect on the effective modulus of concrete.

Keywords: Elastic moduli; Micromechanics; Microstructure; Interfacial transition zone; Particle size distribution

The elastic modulus of concrete has been studied extensively. By assuming the concrete was a two-phase composite material, it was experimentally verified [1] that the test results fell below the lower Hashin and Shtrikman (H-S) bound [2]—a general bound for two-phase composite materials. It was pointed out [1] that the low results in such investigations could be related to the noncompensation of porosity in the specimens. Nilsen and Monteiro [3] and Simeonov and Ahmad [1] used H-S bounds and experimental data to investigate the elastic modulus of concrete. They concluded that concrete should be considered to be a three-phase composite instead of a two-phase one.

It has been known that the structure of the cement paste in the vicinity of the aggregates differs from that of bulk cement paste. The phase in which the presence of the aggregates affects the properties of cement paste is taken as the Interfacial Transition Zone (ITZ) [4,5]. Therefore, a three-phase concrete is a composite material with aggregates as dispersed phase, ITZ as interphase, and cement paste as continuous phase or matrix. In addition to the influence of the elastic modulus and volume fraction of aggregates and cement paste on the elastic modulus of concrete, it has been found by experiment that the elastic modulus of three-phase concrete is intimately related to the elastic modulus and volume fraction of ITZ [1]. Because the volume fraction of ITZ is determined by the surface area of aggregate, while the surface area of aggregate is dependent on the maximum aggregate size and aggregate gradation, the volume fraction of ITZ and the corresponding elastic modulus of concrete should be closely related to the maximum aggregate size and aggregate gradation [6]. Therefore, a reasonable model should be able to consider the influence of the maximum aggregate size and aggregate gradation on the elastic modulus of three-phase concrete.

Neubauer et al. [7], Ramesh et al. [8], and Garboczi [9] have developed models in which the ITZ was represented by a thin and shell-like region that surrounded each aggregate. These three-shell (aggregate/ITZ/cement paste) models should be more realistic than two-component models. A limitation of these models is that they did not consider the effect of the maximum aggregate size and aggregate gradation on the elastic moduli of concrete. Consequently, an improved model is desirable.

For two-phase particulate-filled or fiber-reinforced composite materials, Christensen and Lo [10] developed a three-phase sphere model to estimate the effective bulk modulus and shear modulus. By overall evaluations, Christensen [11] concluded that this three-phase sphere model was more reasonable and reliable than other generally used models, such as the differential scheme and the Mori-Tanaka model [12] because the stress-strain field interactions between different inclusions were considered in this model. Although Christensen and Lo's model has the potential to be extended to three-phase concrete, modifications are needed to make it

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possible to evaluate the effect of the maximum aggregate size and aggregate gradation on the elastic modulus of concrete.

The objective of this study is to extend Christensen and Lo's model to three-phase concrete that is able to consider the effect of the maximum aggregate size and aggregate gradation. Calculated results will be compared with experimental results found in the literature. The influential parameters such as the maximum aggregate size, aggregate gradation, and the thickness and elastic modulus of ITZ will be discussed via calculated results.

1. Formulation development

1.1. Micromechanical model

For two-phase composite materials, Christensen and Lo developed a three-phase sphere model to predict the effective moduli [10]. Taking the inclusion as aggregate and the matrix as cement paste, Christensen and Lo's model can be extended to a four-layer sphere model by adding an ITZ layer in between the inclusion and the matrix layer. This four-phase sphere model is used in this paper to represent three-phase concrete, as shown in Fig. 1. It is noted that the elastic properties of ITZ vary through its thickness. In the proposed four-phase sphere model, however, ITZ is assumed to be a uniform layer for simplicity.

In Fig. 1, letter a denotes the radius of the aggregate, b-a denotes the thickness of ITZ layer, and c-b denotes the thickness of cement paste layer. The shaded infinite area is the equivalent concrete medium. Other physical-mechanical parameters are $E_0(a)$, $K_0(a)$, $G_0(a)$, and $v_0(a)$, which are the effective Young's modulus, bulk modulus, shear modulus, and Poisson's ratio of the equivalent concrete medium, respectively. E_i, K_i, G_i, and v_i ($i = 1, 2, 3$) are the Young's modulus, bulk modulus, shear modulus, and Poisson's ratio of cement paste ($i = 1$), ITZ ($i = 2$), and aggregate ($i = 3$), respectively.

1.2. Formulation development

According to Christensen and Lo [10], if we directly follow the procedure of solving the continuum equations at the interface of $r = a$, $r = b$, and $r = c$ (where r is the spherical polar coordinate), there are 12 variables to be solved simultaneously. Obviously, this procedure is very tedious. Besides, when applying Eshelby's theory to this four-phase sphere model, it is difficult to determine which phase is matrix and which phase is inclusion because of the existence of the ITZ interphase. Therefore, the problem described in Fig. 1 should be further studied.

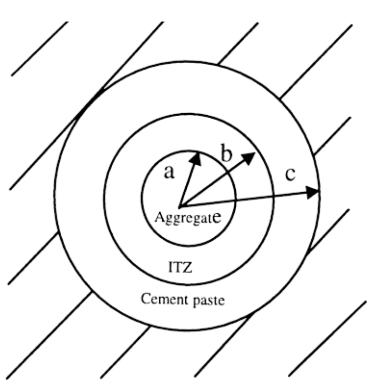

Fig. 1. Four-phase sphere model for three-phase concrete.

Assuming an ITZ-coated aggregate as an "equivalent particle" (i.e., a homogeneous particle with equivalent elastic properties and radius of b), then the four-phase sphere model shown in Fig. 1 is reduced to a three-phase sphere model shown in Fig. 2(a). Taking the equivalent particle as a separated body, it can be represented by a two-phase sphere model shown in Fig. 2(b). Therefore, the four-phase sphere model in Fig. 1 is reduced to the three-phase sphere model in Fig. 2(a) plus the two-phase sphere model in Fig. 2(b). To keep this treatment theoretically sound, the unknown equivalent elastic properties of the equivalent particle in Fig. 2(a) should take such values that the interfacial stress and strain at $r = b$ in Fig. 1 and Fig. 2(a) are the same when the two models are subjected to the same boundary conditions. This can be achieved if the interfacial stress and strain at $r = b$ in Fig. 2(a) are used as the boundary conditions of the two-phase sphere model in Fig. 2(b). Except for the unknown equivalent elastic properties of the equivalent particle, the three-phase sphere model in Fig. 2(a) has been successfully solved for equivalent bulk modulus and shear modulus by Christensen and Lo [10] and Christensen [13]. When solving for the equivalent bulk modulus in Fig. 2(a), Christensen [13] applied an arbitrary hydraulic pressure at the infinite boundary. Under this boundary condition, Fig. 2(a) becomes a spherical symmetry problem. At the surface of the equivalent particle (i.e., at $r = b$), only the interfacial radial stress and displacement are induced. Taking this interfacial radial stress and displacement as the boundary conditions of the two-phase sphere model shown in Fig. 2(b), the equivalent bulk modulus of the equivalent particle can be obtained by solving this model. Again, Christensen [13] has solved for the equivalent bulk modulus represented by Fig. 2(b) by applying either an arbitrary boundary hydraulic pressure or an arbitrary boundary radial displacement. Christensen [13] has proved that the same equivalent bulk modulus is obtained whether an arbitrary radial boundary stress or displacement is applied. This suggests that both the stress and displacement boundaries be satisfied in Fig. 2(b). It is noted that this simplification is an efficient method only for equivalent bulk modulus because of the spherical symmetry stress condition. If, for example, the shear boundary condition is applied to obtain the equivalent shear modulus, the two-phase sphere model described in Fig. 2(b) will become very complicated.

Using the theory of elasticity [14] and Eshelby's equivalent medium theory [15], Christensen [13] proved that the effective bulk modulus for the three-phase sphere model shown in Fig. 2(a) is shown in Eq. (1):

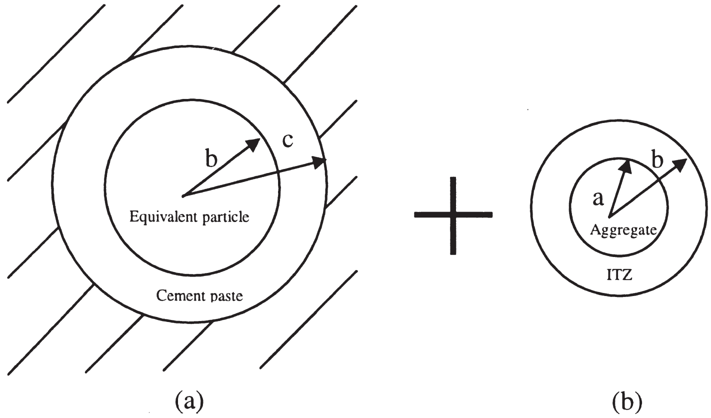

Fig. 2. *(a)* and *(b):* Equivalent three-phase sphere model and two-phase sphere model.

$$K_0(a) = K_1 + \frac{(K_e - K_1)(b^3/c^3)}{1 + \left[(1 - b^3/c^3)(K_e - K_1)\Big/\left(K_1 + \frac{4}{3}G_1\right)\right]} \quad (1)$$

where $K_0(a)$ is the bulk modulus of the three-phase sphere model described by Fig. 2(*a*). The variable *a* in $K_0(a)$ reflects its aggregate size dependent nature. K_e is the equivalent effective bulk modulus of the equivalent particle in Fig. 2(*a*), which will be given by solving the equivalent problem described in Fig. 2(*b*).

For the problem described in Fig. 2(*b*), Christensen [13] derived the effective bulk modulus K_e as shown in Eq. (2):

$$K_e = K_2 + \frac{(K_3 - K_2)a^3/b^3}{1 + (1 - a^3/b^3)\left[(K_3 - K_2)\Big/\left(K_2 + \frac{4}{3}G_2\right)\right]} \quad (2)$$

Substituting Eq. (2) into Eq. (1), the effective bulk modulus for three-phase concrete can be obtained.

It can be seen from Eqs. (1) and (2) that the effective bulk modulus of concrete depends not only on the elastic properties of aggregate, ITZ, and cement paste, but also on the aggregate size, *a*, ITZ layer thickness, *b-a*, and cement paste layer thickness, *c-b*. For a given concrete, the elastic properties of aggregate and cement paste, the maximum aggregate size, and the aggregate gradation are known. The elastic properties of ITZ can be estimated from existing studies. For example, Lutz et al. [16] indicated that the Young's modulus of ITZ is about 30–50% less than the Young's modulus of cement paste. This means K_1, K_2, K_3, G_1, G_2, and *a* are known parameters for a given concrete. However, it is necessary to determine the parameters, *b* and *c*.

It has been shown that in actual concrete, although the thickness of the ITZ layer depends on factors such as water/cement ratio, it seems to be independent of the size of inclusion [17]. Therefore, it is reasonable to assume the thickness of ITZ layer (*b-a*) to be constant regardless of the size distribution of aggregates. It is noted that the ITZ layer may overlap between neighboring aggregates, which depends largely on the initial concrete mixture proportions and the ultimate curing conditions. Garboczi and Bentz [18,19] proposed a multiscale model to estimate the ITZ volume fraction for a given aggregate gradation and ITZ thickness. Their model can consider the overlap effect but needs more algebra. For simplicity, we assume that there is no overlap of ITZ layers between neighboring aggregates. Based on this assumption, the ITZ layer thickness can be estimated as follows.

If the graded aggregates are divided into *N* grades, the average radius for the i^{th} grade is r_i ($i = 1, 2, \ldots, N$), the aggregate volume fraction for i^{th} grade is V_i, then the total surface area for aggregates in the i^{th} grade is as shown in Eq. (3):

$$S_a(i) = \frac{4\pi r_i^2}{\frac{4}{3}\pi r_i^3} V_i \quad (3)$$

Denoting f_3 is the volume fraction of aggregates in concrete, the total surface area for the total *N* grades is shown in Eq. (4).

$$S_a = f_3 \sum_{i=1}^{N} S_a(i) \quad (4)$$

Assuming every aggregate is coated with the same thickness of ITZ, the ITZ thickness ($b - a$) is shown in Eq. (5):

$$b - a = \frac{f_2}{S_a} \quad (5)$$

where f_2 is the volume fraction of ITZ.

Substituting Eqs. (3) and (4) into Eq. (5), the ITZ thickness is obtained as shown in Eq. (6):

$$b - a = f_2\Big/\left(3 f_3 \sum_{i=1}^{N} \frac{V_i}{r_i}\right) \quad (6)$$

Eq. (6) contains two unknown variables, *b* and f_2, for a given concrete. Before Eq. (6) can be used, either *b* or f_2 should be determined. Also, it is noted that the number of grades, *N*, in Eq. (6), should be large enough to ensure accuracy.

Garbozci and Bentz [19] developed a mapping approach to determine the uniform ITZ layer thickness. Using this approach, the dilute limit of an aggregate surrounded by a gradient in properties can be mapped into the case of an aggregate surrounded by a shell having a given thickness and uniform property. It is noted that the uniform layer thickness and the elastic modulus should match (i.e., the uniform ITZ layer thickness depends on what quantity is chosen for its elastic modulus). As the ITZ layer thickness is increased, a larger elastic modulus should be used [19]. However, such a quantitative relation is unavailable at present. This will be a subject of further research. Actually, a number of researchers have experimentally investigated the ITZ layer thickness and elastic modulus. They found that the average thickness of ITZ layer in typical concrete is about 0.05 mm and the elastic modulus is about 30–50% less than the elastic modulus of cement paste [4,5,16,17]. Assuming the term $b - a$ in Eq. (6) is about 0.05 mm, the ITZ volume fraction, f_2, can be obtained using Eq. (6) as shown in Eq. (7):

$$f_2 = 0.15 f_3 \sum_{i=1}^{N} \frac{V_i}{r_i} \tag{7}$$

The volume fractions of aggregate, ITZ, and cement paste have the relation shown in Eq. (8):

$$f_1 + f_2 + f_3 = 1 \tag{8}$$

in which f_1 is the volume fraction of cement paste.

Once the volume fraction of ITZ, f_2, is determined from Eq. (7), the volume fraction of cement paste, f_1, can be derived from Eq. (8).

The unknown parameter, c, can be obtained by applying Christensen and Lo's formulations of two-phase composite materials as shown in Fig. 2(*a*). Christensen and Lo [10] assumed that the ratio of the radius of the particle to the radius of the particle plus the thickness of matrix layer is a constant—$f^{1/3}$, where f is the volume fraction of particles. This configuration requires that the particles have a distribution with particle sizes varying down to infinitesimally small. Using Christensen and Lo's assumptions, the ratio of b/c in Fig. 2(*a*) is a constant, and the relation in Eq. (9) shows:

$$c = b/(f_2 + f_3)^{1/3} \tag{9}$$

From Eq. (9), the unknown parameter c can be determined because b, f_2, and f_3 are known parameters.

It can be seen from Eqs. (1) through (9) that once the aggregate radius, a, is specified, the parameters b, c, and the effective bulk modulus of concrete, $K_0(a)$, can be determined. Because the aggregates are graded, the number of aggregates in concrete is infinite. Since each aggregate particle contributes a bit to the bulk modulus of the whole mix, the equivalent bulk modulus of the three-phase concrete, K_0, is shown in Eq. (10):

$$K_0 = \int_{a_{\min}}^{a_{\max}} K_0(a)\,dP(a) \tag{10}$$

where $a_{\min}$ and $a_{\max}$ are the minimum aggregate radius and maximum aggregate radius, respectively; while $P(a)$ is the percent passing (by mass) of aggregates (i.e., the mass fraction of all the aggregates with radius smaller than the specified value, a). If aggregates of different size have all the same density, the mass fractions are the same as volume fractions.

In general, aggregate gradation in concrete is loosely restricted. In order to consider the effect of aggregate gradation on the effective bulk modulus by the present model, the relation shown in Eq. (11) for continuously graded aggregates is used as an example:

$$P(a) = \left(\frac{a}{a_{\max}}\right)^M \tag{11}$$

where M is a parameter determining the shape of the gradation curve. Theoretically, if $M \leq 0.5$, the larger the M value, the greater the coarse aggregates [20]. Fig. 3 shows two gradation curves. In Fig. 3, gradation with $M = 0.5$ contains more coarse aggregates than gradation with $M = 0.4$.

Due to the complicated integration, Eq. (10) is replaced by Eq. (12) for simplicity:

$$K_0 = \sum_{i=1}^{N} K_0(r_i) V_i \tag{12}$$

where r_i and V_i are the same as in Eq. (3). V_i is dependent on aggregate gradations. If the aggregate gradation is represented by Eq. (11), then $V_i = P(r_{i+1}) - P(r_i)$.

Eq. (12) will be used for calculation in the present paper. When conducting calculations, the step for the aggregate radius, $\Delta r_i = r_{i+1} - r_i$, is assumed as $\Delta r_i = 0.001 a_{\max.}$

In practice, Young's modulus is often provided by experiments. In order to compare the results predicted by the four-phase sphere model with the experimental results, it is desirable to solve for the Young's modulus of concrete.

Macroscopically, concrete is an isotropic material. For isotropic elastic bodies, there are four elastic constants (i.e.,

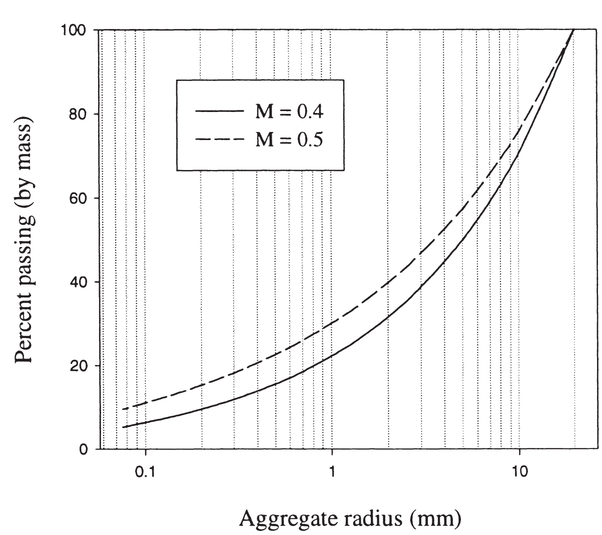

Fig. 3. Aggregate gradations.

Young's modulus, E, bulk modulus, K, shear modulus, G, and Poisson's ratio, ν). These constants posses the relationships shown in Eqs. (13) and (14) [14]:

$$E = 9K(1-2\nu)/2 \tag{13}$$

$$G = 9K(1-2\nu)/(3-4\nu) \tag{14}$$

It can be seen from Eqs. (13) and (14) that, for the four elastic constants, only two are independent parameters. Therefore, if either the Poisson's ratio or the shear modulus is provided, the Young's modulus of concrete can be solved by Eqs. (13) and (14).

If we directly solving for the effective shear modulus from the four-phase sphere model shown in Fig. 1, the process will be very complicated according to Christensen and Lo [10]. In addition, the derived result is inconvenient to use because it will be in the form of a quadric equation. Compared with the shear modulus of concrete, the Poisson's ratio varies slightly. Therefore, Poisson's ratio can be estimated through approximate approach. Because there are no formulas available to estimate the Poisson's ratio of concrete, a simplified rule of mixture approach will be used.

Assuming aggregate phase, ITZ layer, and cement paste phase is in series in transverse direction, the total transverse strain, ε_0, is the sum of the transverse strain of cement paste phase, ε_1, ITZ layer, ε_2, and aggregate, ε_3. Taking the volume fraction of each phase, f_1, f_2, and f_3 as the weighting factors, the relation in Eq. (15) holds:

$$\varepsilon_0 = f_1\varepsilon_1 + f_2\varepsilon_2 + f_3\varepsilon_3 \tag{15}$$

assuming each phase in concrete is isostrain in axial direction (i.e., each phase has the same axial strain, ε). According to the definition of Poisson's ratio (i.e., the negative of the ratio of transverse strain to axial strain), the effective Poisson's ratio of concrete is estimated by dividing both sides of Eq. (15) by ε, as shown in Eq. (16):

$$\nu_0 = f_1\nu_1 + f_2\nu_2 + f_3\nu_3 \tag{16}$$

Simultaneously solving Eqs. (1) through (16), the Young's modulus, E_0, and shear modulus, G_0, of three-phase concrete can be obtained. The calculation process is completed via a Fortran program.

2. Results and discussions

Although a lot of test results for Young's modulus of concrete are available in the literature, most of them do not provide detailed information. Therefore, only a few are suitable for purpose of comparison. Stock et al. [21] have provided detailed information except for the Poisson's ratio, Young's modulus, and layer thickness of the ITZ. In order to compare the test results with the results predicted by the present model, these parameters should be assumed. According to Simeonov and Ahmad [1], the Poisson's ratio for each phase can be taken as: cement paste, $\nu_1 = 0.25$; ITZ, $\nu_2 = 0.3$; and aggregate, $\nu_3 = 0.15$. According to Lutz et al. [16], the Young's modulus of ITZ is $E_2 = 0.4E_1$, where E_1 is the Young's modulus of cement paste. The ITZ thickness is assumed as 0.05 mm [3,5,17]. In the experiments, Stock et al. [21] used a maximum aggregate radius of $a_{max} = 9.5$ mm, a minimum aggregate radius of $a_{min} = 0.15$ mm, and an aggregate gradation of grading curve 3 in British Road Research Laboratory Road Note No. 4. Table 1 shows the test results by Stock et al. and the predicted result by the proposed four-phase sphere model.

In Table 1, when the aggregate content is in the range of practical concrete ($f_3 = 0.4 – 0.8$), the difference between the test results and the predicted results is small, suggesting that the proposed model is suitable for the elastic modulus estimation of practical concrete. When the volume fraction of aggregate is small ($f_3 = 0.2$), the difference between the predicted and test results are comparatively large. Theoretically, the proposed model should work better at small volume fractions of aggregates because the effective medium theories are based on exact dilute results and Eq. (6) is a good approximation at $f_3 = 0.2$. The comparatively large deviation at $f_3 = 0.2$ may be due to some other reasons. One possible reason may be that the isostrain assumption in Eq. (16) may not be valid for small aggregate concentration. This is because, in this case, cement paste and ITZ layer bear load comparable to that by aggregate due to their large volume fractions. Usually, the elastic moduli of cement paste and ITZ layer are much smaller than that of aggregate. Therefore, the comparable load and the incomparable elastic modulus between either cement paste or ITZ and aggregate will result in a strain condition different from isostrain.

To evaluate the effect of the maximum aggregate size and aggregate gradation on the elastic modulus of concrete, calculations are conducted by using the four-phase sphere model. When conducting calculations, the parameters used are as follow: $E_1 = 15$ GPa, $E_2 = 7.5$ GPa, $E_3 = 70$ GPa, $f_3 = 0.6$, $\nu_1 = 0.25$, $\nu_2 = 0.3$, and $\nu_3 = 0.15$. ITZ thickness is assumed as 0.05 mm. Aggregate gradation is assumed as in Eq. (11), with $M = 0.5$ as gradation 1 and $M = 0.4$ as gradation 2. Calculated results are shown in Fig. 4.

Table 1
Test and predicted results of Young's modulus (GPa)

Volume fraction of aggregate	Test (compression) (1)	Predicted (2)	Deviation (%) [(1) − (2)]/(1)	Test (tension) (3)	Predicted (4)	Deviation (%) [(3) − (4)]/(3)
0.2	17.80	15.88	10.79	15.80	17.46	−10.51
0.4	21.40	20.69	3.32	23.20	22.98	0.94
0.6	29.00	28.33	2.31	30.70	30.68	0.06
0.8	41.30	40.17	2.73	39.10	41.86	−7.06

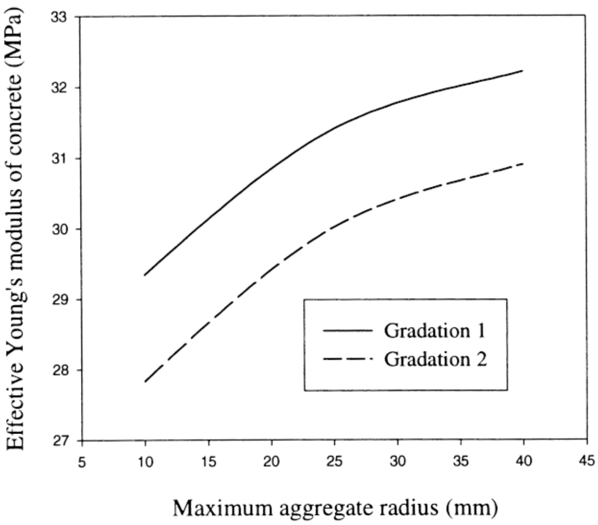

Fig. 4. Effect of the maximum aggregate size and aggregate gradation on the effective Young's modulus of concrete.

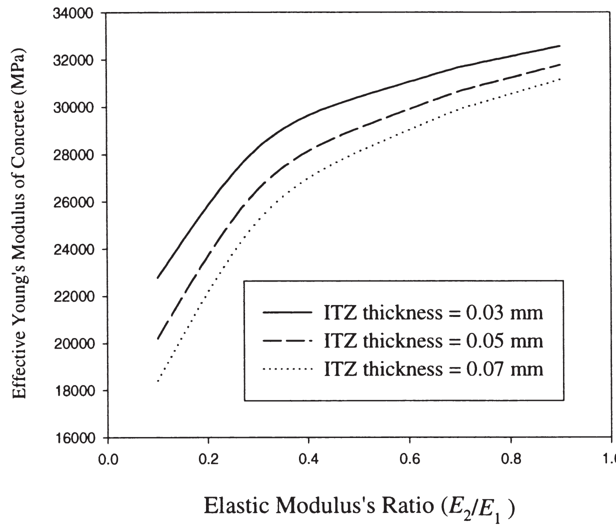

Fig. 5. Effect of the elastic modulus and thickness of ITZ layer on the effective Young's modulus of concrete.

In Fig. 4, the Young's modulus of concrete increases as the maximum aggregate size increases. The trend of this result is supported by an experiment [6]. The reason for this result may be that the aggregate surface area decreases as coarser aggregates increase, which will reduce the volume fraction of ITZ. Because the elastic modulus of ITZ is low, the decrease of the volume fraction of ITZ will make the elastic modulus of concrete increase. It can be seen from Fig. 3 that gradation 1 contains more coarser aggregates than gradation 2; thus aggregates with gradation 1 will have smaller surface area than that with gradation 2. Consequently, concrete with aggregate gradation 1 has higher elastic modulus than that with gradation 2. The result in Fig. 4 is in agreement with this analysis.

The thickness and the elastic properties of the ITZ layer depend on the initial concrete mixture proportions and the ultimate curing conditions [18]. Therefore, they are not constants. The effect of the elastic modulus and the thickness of the ITZ layer on the effective Young's modulus of concrete is shown in Fig. 5. To obtain Fig. 5, an aggregate gradation shown in Eq. (11) with $M = 0.5$ was used. The maximum aggregate radius was 20 mm. In Fig. 5, the effective Young's modulus of concrete increases as the elastic modulus of ITZ layer increases. The effect is more distinct when the elastic modulus of ITZ layer is comparatively small, such as when $E_2/E_1 < 0.4$. Therefore, increasing the elastic modulus of ITZ layer through approaches such as using silica fume is efficient in increasing the effective Young's modulus of concrete. Also, the effective Young's modulus of concrete increases as the thickness of the ITZ layer decreases. Again, the effect becomes large when the elastic modulus of ITZ layer is small. In actual concrete, reducing the ITZ layer thickness through approaches such as decreasing the water/cement ratio is also efficient in enhancing the rigidity of concrete.

3. Conclusions

An approach to evaluate the effect of the maximum aggregate size and aggregate gradation on the effective bulk modulus of three-phase concrete was proposed by extending Christensen and Lo's three-phase sphere model to a four-phase sphere model. Formulations were developed based on several simplified assumptions. With the help of the theory of elasticity and the simplified rule of mixture for Poisson's ratio, the effective Young's modulus was also obtained. Based on the calculated results by using this four-phase sphere model and the test results by Stock et al., the following conclusions are reached:

1. The calculated Young's modulus is close to the test result, particularly when the volume fraction of aggregates is within its usually used range. This comparison result suggests that the proposed formulations can be used to estimate the effective elastic moduli of actual concrete.
2. The proposed model can characterize the effect of the maximum aggregate size and aggregate gradation on the effective moduli of concrete. The calculated result suggests that the maximum aggregate size and aggregate gradation have a significant effect on the effective moduli of concrete. Increasing the maximum aggregate size and using denser aggregate gradation can increase the effective moduli of concrete.
3. The ITZ layer has a distinct effect on the effective elastic moduli of concrete. Approaches such as using silica fume to increase the elastic modulus of ITZ layer or reducing the water/cement ratio to decrease the ITZ layer thickness are efficient in increasing the effective elastic moduli of concrete.
4. An advantage of the proposed model for predicting the effective moduli of concrete is its simplicity, which makes it easy to use in practical applications.

It is noted that several assumptions were made in the present paper. It is the focus of further studies to refine the results obtained in this paper. In addition, systematic experiments are needed to rigorously validate the effect of the maximum aggregate size and aggregate gradation on the effective elastic moduli of concrete.

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