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# EFFECT OF TRANSITION ZONE ON ELASTIC MODULI OF CONCRETE MATERIALS

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## ABSTRACT

This paper proposes a composite model for evaluation of the elastic moduli of concrete materials. Concrete or mortar is assumed to have three phases, namely: aggregate, matrix and transition zone. Analytical solutions for effective elastic moduli of the proposed composite model are presented. The effect of the elastic moduli and volume fraction of the transition zone on overall elastic moduli of concrete or mortar is investigated. A procedure is also suggested to determine the elastic moduli of the transition zone knowing the overall elastic moduli of concrete or mortar.

## Introduction

The primary phases in concrete or mortar are aggregate and matrix (bulk paste). Experiments have shown that the matrix surrounding the aggregate has quite different stiffness properties than that away from it. Thus, a correct description of micro-structure of the concrete requires the recognition of the third important phase namely the *transition zone* or *interphase* [1]. The mechanical properties of concrete are controlled by the presence of the transition zone. The transition zone has a relatively more porous structure than that of the matrix. The boundary between transition zone and matrix is hard to distinguish, and its stiffness may vary throughout its thickness.

It is estimated from experiments that the transition zone has lower stiffness than the bulk paste [2]. Thus, if the volume of the transition zone is not negligible, this may significantly affect the overall elastic moduli of mortar or concrete. Also, as pointed out by Cohen et al. [2], the addition of admixtures such as silica fume increases the stiffness of the transition zone and thus improves the overall stiffness of concrete. Nilsen and Monteiro [3] have shown that the transition zone has a significant effect on the overall elastic moduli of concrete and mortar. The strength of concrete is limited by the presence of such transition zone, which is the weakest link in the composite system. Thus, in order to realistically characterize concrete or mortar materials, the transition zone must be incorporated in its micro-mechanical model.

As pointed out by Nilsen and Monteiro [3], a lot of research in concrete mechanics has already been done to assess the influence of transition zone on overall strength of concrete. However, little work has been done to predict the effect of the stiffness and volume of transition zone on the overall stiffness of concrete. Some of the early mechanics based models used for the estimation of the elastic moduli of concrete are: Reuss [4] or series model, Voigt [5] or parallel model. Hirsch [6], Counto [7], Lydon and Balendran [8] among others suggested empirical expressions for the estimation of elastic moduli of mortar or concrete.

In the field of composite mechanics, Hashin and Shtrikman [9] have derived bounds for elastic moduli of two phase materials. Christinsen and Lo [10,11] have obtained exact solutions for elastic moduli of a three phase composite model consisting of inclusion, matrix and equivalent homogeneous medium. Herve and Zaoui [12] have extended Christinsen's method for a composite model having *n*-phases. Mandel and Dantu [13] experimentally verified Hashin-Shtrikman (H-S) bounds for cement composites. Monteiro [14] suggested that H-S bounds can be used to asses the significance of transition zone on overall elastic moduli. Nilsen and Monteiro [3] analyzed Hirsch's data [6] by evaluating H-S bounds. Simeonov and Ahmed [15] have also used H-S bounds to assess the importance of the transition zone on overall elastic moduli of concrete or mortar. Zhou et al. [16] have studied the effect of coarse aggregate on the elastic moduli of concrete. Lutz and Monteiro [17] have considered the effect of variation of stiffness of the transition zone throughout its thickness in estimating the overall elastic moduli of concrete. As suggested by Nilsen and Monteiro [3], the issue about the recognition of transition zone in elastic moduli estimation can be resolved by evaluating H-S bounds for concrete. However, to *determine* the overall elastic moduli (of concrete and mortar), some assumptions about the shapes of inclusion and transition zone in a composite model are needed.

In this paper, a *four phase composite model* consisting of aggregate, matrix, transition zone and equivalent homogeneous medium is used to model a three phase composite such as concrete or mortar. Analytical solutions for the effective elastic moduli of the four phase composite model are presented. In the present work, this composite model is used to determine the elastic moduli of concrete materials. A discussion on appropriateness of the assumptions of this composite model used for the determination of elastic moduli of concrete materials is presented. The effect of stiffness and volume fraction of transition zone on the overall elastic moduli of concrete or mortar is investigated. A procedure is also suggested for determining the elastic moduli of the transition zone knowing the overall elastic moduli of concrete or mortar.

## Modeling of Problem and Assumptions

Hashin [18] proposed a two phase composite spheres model and derived analytical solutions for its effective elastic moduli. This composite spheres model is extended in the present work for a material having three phases as shown in Fig.1. The dotted lines in this figure denote the boundary of the transition zone. The broken lines in Fig.1 show the imaginary matrix associated with each aggregate and transition zone.

Following Christinsen and Lo [10], except for one individual composite sphere of three phases all other spheres are replaced by an equivalent homogeneous medium. The resulting *four phase composite model* is shown in Fig.2. The material boundaries are distinguished by the *characteristic radii* $a$, $b$ and $c$. The properties $k$, $\mu$ and $\nu$ for each phase is indicated by the respective subscript (1 for inclusion, 2 for transition zone, 3 for matrix and 4 for equivalent

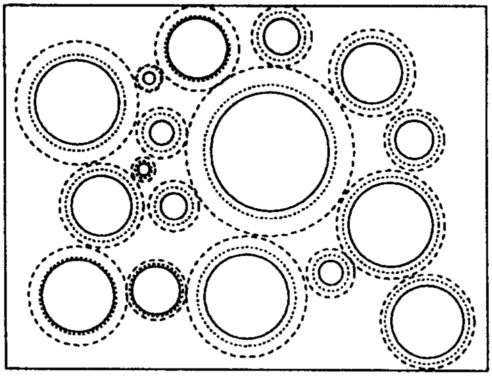

FIG. 1.
Composite spheres model of present work

homogeneous medium). The aggregate with associated transition zone and matrix can be viewed as a *heterogeneous inclusion* that is embedded in the infinite equivalent homogeneous medium. The assumptions used in analytical solutions of this model are detailed below.

1. The elastic moduli of constituent phases and also those of resulting composite are assumed to be isotropic.

2. The aggregate inclusions are assumed to be spherical. In concrete, the aggregates usually have randomly varying shapes. The sharp corners of the aggregates induce stress concentrations that initiate micro-crack propagation. The overall stiffness of a composite unlike strength is a *average macroscopic property*, and is less sensitive to the shape of the inclusion.

3. The volume fraction of the inclusion ($(a/c)^3$) and the volume fraction of the transition zone ($(b^3 - a^3)/c^3$) are assumed to be constant for each inclusion. This assumption indicates that the ratios $a/c$ and $b/c$ are constant for an inclusion independent of its

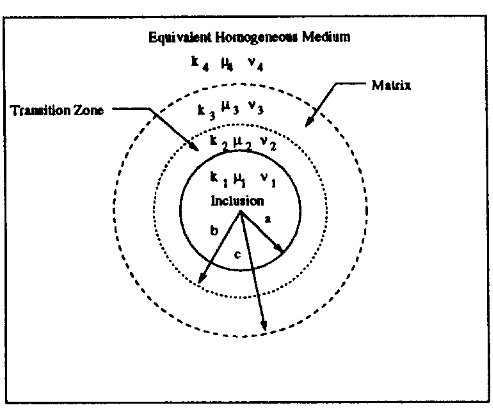


FIG. 2.
Four phase composite model

absolute size. Thus, this assumption enables one to determine the ratios $a/c$ and $b/c$ knowing the volume fractions of inclusion and transition zone (as these ratios are *same* for any inclusion). Also, as all the heterogeneous inclusions with associated transition zones should still have a volume filling configuration, this also implies that the particle size distribution should vary down to the infinitesimal. Hence the theory used for analytical solutions is not appropriate for composites having uniform sized inclusions. In concrete, aggregates of uniform sizes are seldom used and a proper gradation of aggregates is important for decreasing the void space (thus reducing the cement paste content).

4. The derivation of analytical solutions use linear theory of elasticity. The effects of porosity in transition zone, aggregate and matrix are not considered.

5. The theory used for analytical solutions neglects the mutual interaction energy of heterogeneous inclusions (of aggregate, matrix and transition zone) embedded in the equivalent homogeneous medium.

6. The thickness of transition zone is assumed to be constant. Also, its elastic moduli are taken to be constant throughout its thickness.

In the case of concrete or mortar, the transition zone is always 'naturally' associated with the aggregate. This justifies the application of a composite model such as that shown in Fig.2 for determining elastic moduli of materials like concrete. On the other hand, if the transition zone were not to be associated with the aggregate, this problem could have been a more general three phase composite problem, which is relatively difficult to analyze.

## Analytical Solutions for Effective Elastic Moduli of Four Phase Composite Model

This section describes the analytical solutions for the effective elastic moduli of the composite model shown in Fig.2. The determination of the isotropic effective elastic moduli for this model requires the application of appropriate stress or displacement conditions as explained below.

Bulk Modulus Evaluation. To evaluate the bulk modulus, the four phase composite model is subjected to radial pressure or radial displacement conditions at infinite distance from the origin. In spherical polar coordinates this problem reduces to a simple one dimensional problem. From the general solution of this problem, the displacement and stress solutions in various phases can be assumed. The satisfaction of displacement and stress continuity conditions at the inter-material boundaries results in the following equations.

$$A_1 a = A_2 a + B_2 a^{-2} \quad (1)$$

$$3k_1 A_1 = 3k_2 A_2 - 4\mu_2 B_2 a^{-3} \quad (2)$$

$$A_2 b + B_2 b^{-2} = A_3 b + B_3 b^{-2} \quad (3)$$

$$3k_2 A_2 - 4\mu_2 B_2 b^{-3} = 3k_3 A_3 - 4\mu_3 B_3 b^{-3} \quad (4)$$

$$A_3 c + B_3 c^{-2} = A_4 c + B_4 c^{-2} \quad (5)$$

$$3k_3 A_3 - 4\mu_3 B_3 c^{-3} = 3k_4 A_4 - 4\mu_4 B_4 c^{-3} \quad (6)$$

There are seven constants $A_1$, $A_2$, $B_2$, $A_3$, $B_3$, $A_4$, $B_4$ in equations (1)—(6) in addition to unknown parameters $k_4$ and $\mu_4$ of the equivalent homogeneous medium. An additional equation is obtained following the criterion suggested by Christinsen and Lo [10] and is explained next. Under the condition of applied radial pressure, the strain energy in the composite model from the Eshelby formula [19] can be written as

$$U_{comp} = U_o + \frac{1}{2}\int_0^{2\pi}\int_0^{\pi} \left(\sigma_{rr}^o u_r - \sigma_{rr} u_r^o\right) c^2 \sin\theta \, d\theta \, d\phi \tag{7}$$

The inclusion with transition zone and matrix can be treated as a heterogeneous inclusion embedded in an infinite equivalent homogeneous medium. In the Eshelby formula (7), the strain energy $U_o$ is the strain energy of the medium composed only of matrix, in this case the equivalent homogeneous medium. Also, in equation (7), $U_{comp}$ refers to the strain energy of the equivalent homogeneous medium. Thus $U_{comp} = U_o$, and this result is substituted in equation (7) leading to the following conclusion [20].

$$B_4 = 0 \tag{8}$$

Substituting this condition in equations (1)—(6), the resulting *homogeneous* set of equations involve six constants $A_1$, $A_2$, $B_2$, $A_3$, $B_3$, $A_4$ and also the unknown bulk modulus of the equivalent homogeneous medium $k_4$. The nontrivial solution of this set of equations is only possible if and only if the determinant of the matrix containing the coefficients of constants is zero. This condition gives rise to a *linear* equation in terms of $k_4$, from which the expression for $k_4$ can be obtained. Defining,

$$\gamma_1 = (3k_1 + 4\mu_2) b^3 \left[k_3 (4\mu_3 + 3k_2) c^3 + 4\mu_3 (k_2 - k_3) b^3\right] \tag{9}$$

$$\gamma_2 = 4 (k_1 - k_2) a^3 \left[3k_3 (\mu_2 - \mu_3) c^3 + \mu_3 (3k_3 + 4\mu_2) b^3\right] \tag{10}$$

$$\gamma_3 = (3k_1 + 4\mu_2) b^3 \left[(4\mu_3 + 3k_2) c^3 - 3 (k_2 - k_3) b^3\right] \tag{11}$$

$$\gamma_4 = 3 (k_1 - k_2) a^3 \left[4 (\mu_2 - \mu_3) c^3 - (3k_3 + 4\mu_2) b^3\right] \tag{12}$$

The expression for $k_4$ is given by

$$k_4 = \frac{\gamma_1 + \gamma_2}{\gamma_3 + \gamma_4} \tag{13}$$

It should also be noted that the same expression for $k_4$ could have been obtained by applying the radial displacement, for which case the equation (7) is slightly different. In reference [20] it is also shown that the expression for bulk modulus obtained for the four phase composite model is *identical* to that of the composite sphere consisting of three phases subjected to radial pressure/displacement. This reinforces the conclusion by Christinsen and Lo [10], that the bulk modulus solution for their three phase model is identical to that of composite spheres model consisting of two phases.

Shear Modulus Evaluation. To evaluate the shear modulus, the four phase composite model is subjected to shearing traction or shearing displacement at infinite distance from the origin. The assumed stress and displacement solutions in each phase are described in detail in reference [20]. The satisfaction of inter-material boundary displacement and stress continuity conditions give rise to the following set of equations.

$$A_1 a - \Gamma_1(1) A_2 a^3 = B_1 a - \Gamma_1(2) B_2 a^3 + 3B_3 a^{-4} + \Gamma_3(2) B_4 a^{-2} \tag{14}$$

$$A_1 a - \Gamma_2(1) A_2 a^3 = B_1 a - \Gamma_2(2) B_2 a^3 - 2B_3 a^{-4} + 2B_4 a^{-2} \tag{15}$$

$$\begin{aligned} 21\lambda_1 A_2 a^2 + 2\mu_1 \left[A_1 - 3\Gamma_1(1) A_2 a^2\right] \\ = \lambda_2 \left(21 B_2 a^2 - 6B_4 a^{-3}\right) + 2\mu_2 \left[B_1 - 3\Gamma_1(2) B_2 a^2 \right. \\ \left. - 12 B_3 a^{-5} - 2\Gamma_3(2) B_4 a^{-3}\right] \end{aligned} \tag{16}$$

$$\mu_1\left[A_1 - \Gamma_4(1)A_2a^2\right] = \mu_2\left[B_1 - \Gamma_4(2)B_2a^2 + \Gamma_5(2)B_4a^{-3} + 8B_3a^{-5}\right] \quad (17)$$

$$B_1b - \Gamma_1(2)B_2b^3 + 3B_3b^{-4} + \Gamma_3(2)B_4b^{-2} = D_1b - \Gamma_1(3)D_2b^3 + 3D_3b^{-4} + \Gamma_3(3)D_4b^{-2} \quad (18)$$

$$B_1b - \Gamma_2(2)B_2b^3 - 2B_3b^{-4} + 2B_4b^{-2} = D_1b - \Gamma_2(3)D_2b^3 - 2D_3b^{-4} + 2D_4b^{-2} \quad (19)$$

$$\lambda_2\left(21B_2b^2 - 6B_4b^{-3}\right) + 2\mu_2\left[B_1 - 3\Gamma_1(2)B_2b^2 - 12B_3b^{-5} - 2\Gamma_3(2)B_4b^{-3}\right] = \lambda_3\left(21D_2b^2 - 6D_4b^{-3}\right) + 2\mu_3\left[D_1 - 3\Gamma_1(3)D_2b^2 - 12D_3b^{-5} - 2\Gamma_3(3)D_4b^{-3}\right] \quad (20)$$

$$\mu_2\left[B_1 - \Gamma_4(2)B_2b^2 + \Gamma_5(2)B_4b^{-3} + 8B_3b^{-5}\right] = \mu_3\left[D_1 - \Gamma_4(3)D_2b^2 + \Gamma_5(3)D_4b^{-3} + 8D_3b^{-5}\right] \quad (21)$$

$$D_1c - \Gamma_1(3)D_2c^3 + 3D_3c^{-4} + \Gamma_3(3)D_4c^{-2} = F_1c + 3F_3c^{-4} + \Gamma_3(4)F_4c^{-2} \quad (22)$$

$$D_1c - \Gamma_2(3)D_2c^3 - 2D_3c^{-4} + 2D_4c^{-2} = F_1c - 2F_3c^{-4} + 2F_4c^{-2} \quad (23)$$

$$\lambda_3\left(21D_2c^2 - 6D_4c^{-3}\right) + 2\mu_3\left[D_1 - 3\Gamma_1(3)D_2c^2 - 12D_3c^{-5} - 2\Gamma_3(3)D_4c^{-3}\right] = \lambda_4\left(-6F_4c^{-3}\right) + 2\mu_4\left[F_1 - 12F_3c^{-5} - 2\Gamma_3(4)F_4c^{-3}\right] \quad (24)$$

$$\mu_3\left[D_1 - \Gamma_4(3)D_2c^2 + \Gamma_5(3)D_4c^{-3} + 8D_3c^{-5}\right] = \mu_4\left[F_1 + \Gamma_5(4)F_4c^{-3} + 8F_3c^{-5}\right] \quad (25)$$

where

$$\Gamma_1(i) = \frac{6\nu_i}{1 - 2\nu_i};\Gamma_2(i) = \frac{7 - 4\nu_i}{1 - 2\nu_i} \quad (26)$$

$$\Gamma_3(i) = \frac{5 - 4\nu_i}{1 - 2\nu_i};\Gamma_4(i) = \frac{7 + 2\nu_i}{1 - 2\nu_i} \quad (27)$$

$$\Gamma_5(i) = \frac{2(1 + \nu_i)}{1 - 2\nu_i} \quad (28)$$

and $i = 1, 2, 3, 4$ for inclusion, transition zone, matrix and equivalent homogeneous medium respectively. Also $\lambda_i$ in above equations denote one of the Lamé parameters of the $i$ $th$ phase.

There are thirteen constants $A_1, A_2, B_1, B_2, B_3, B_4, D_1, D_2, D_3, D_4, F_1, F_3$ and $F_4$ in addition to the unknown properties of the equivalent homogeneous medium ($\lambda_4, \mu_4, \nu_4$) in twelve equations (14)—(25). An additional equation for determination of these constants is obtained by using the Eshelby formula for strain energy in the composite model as follows.

$$U_{comp} = U_o + \frac{1}{2}\int_0^{2\pi}\int_0^{\pi}\left(\sigma^o_{rr}u_r + \sigma^o_{r\theta}u_\theta + \sigma^o_{r\phi}u_\phi - \sigma_{rr}u^o_r - \sigma_{r\theta}u^o_\theta - \sigma_{r\phi}u^o_\phi\right)c^2\sin\theta\, d\theta\, d\phi \quad (29)$$

The similar reasoning discussed in the case of bulk modulus evaluation $\left(U_{comp} = U_o\right)$ leads to the following conclusion [20].

$$F_4 = 0 \quad (30)$$

Substituting (30) in equations (14) —(25), it can be seen that the resulting equations have twelve constants (also unknown shear modulus $\mu_4$ of the equivalent homogeneous medium).

These set of equations form a system of *homogeneous* equations. As explained in the case of bulk modulus, for unique solution of this system of equations, the determinant of the matrix containing coefficients of these constants must be zero. This yields a *quadratic* equation for the shear modulus $\mu_4$. The expression for shear modulus so obtained is complicated and difficult to simplify. Hence, another approach is adopted in the present work, which is explained below.

For any given problem the values of $k_1, \mu_1, \nu_1, k_2, \mu_2, \nu_2, k_3, \mu_3, \nu_3$ are known beforehand. These values are explicitly substituted in the equations (14)—(25), which now have only one unknown $\mu_4$. The determinant of the coefficient matrix can now be evaluated *symbolically*. In the present work the Mathematica software [21] is used to obtain the symbolic equation. The result is a quadratic equation of the form

$$\psi_1 \mu_4^2 + \psi_2 \mu_4 + \psi_3 = 0 \tag{31}$$

In the above equation $\psi_1$, $\psi_2$ and $\psi_3$ are numerical constants that can be evaluated for a given problem. The elastic moduli of a composite material are positive definite and unique. Thus, the non-negative root of the above equation gives the value of $\mu_4$.

Due to the overall isotropy of the composite, knowing $k_4$ and $\mu_4$, the Young's modulus $E_4$ of the equivalent homogeneous medium can be evaluated as

$$E_4 = \frac{9 k_4 \mu_4}{3 k_4 + \mu_4} \tag{32}$$

In reference [20], the derivations for analytical solutions of effective elastic moduli of a two dimensional four phase composite model are also presented.

## Comparison with Experimental Data

The elastic moduli computed from four phase composite model are compared with the experimental data for elastic moduli of concrete and mortar samples reported by Hirsch [6]. In all the computations, due to the absence of experimental data, the volume fraction of transition zone is assumed as 10%, its $E$ value is taken as half of that of the matrix and the $\nu$ of the transition zone is assumed to be same as that of the matrix. The elastic moduli of constituent phases are given in Table 1. The comparison of measured elastic moduli and those computed from four phase composite model is given in Table 2. The second column of Table 2 shows the volume fraction of the aggregate and the third column indicates the experimentally measured value of Young's modulus ($E_d$). It can be noted from Table 2 that $E_4$ from the four phase composite model matches well with experimental data for the aggregate types of gravel, limestone and lead. For these aggregate types, Nilsen and Monteiro [3] reported that experimental results fall outside the H-S bounds. Thus, this may imply that the effect of transition zone needs to be considered only for these aggregate types. Also, a better correlation with experimental results would have been obtained using the four phase composite model by assuming a lesser transition zone volume fraction for steel and glass aggregates. It is also found in this numerical study that the prediction from the four phase composite model is also sensitive to the Poisson's ratio of the constituent phases. It should be however noted that this example illustrates only the application of the four phase composite model and more experimental work on the properties and volume fraction of the transition zone are certainly needed to rigorously validate this model.

## Parametric Studies

Assessment of the Effect of Volume Fraction of Transition Zone on Elastic Moduli of Concrete. The objective this study is to estimate analytically the effect of the thickness of

TABLE 1

Elastic Moduli of Constituent Phases

| Abbreviation | Aggregate Type | E (Gpa) | $\nu$ |
|---|---|---|---|
| ST | Steel | 206 | 0.30 |
| SD | Sand | 75.8 | 0.15 |
| GL | Glass | 73.1 | 0.25 |
| GR | Gravel | 59.6 | 0.15 |
| LI | Limestone | 30.7 | 0.15 |
| PB | Lead | 16.6 | 0.45 |
| CM | Cement | | |
| | Mortar | 19.17 | 0.26 |
| | Transition Zone | 9.59 | 0.26 |

TABLE 2

Comparison of Results

| Batch | $v_1$ | $E_d$ (Gpa) | $k_4$ (Gpa) | $\mu_4$ (Gpa) | $E_4$ (Gpa) |
|---|---|---|---|---|---|
| ST-5 | 0.5 | 53.4 | 26.42 | 16.39 | 40.74 |
| ST-4 | 0.4 | 44.5 | 22.08 | 13.51 | 33.66 |
| ST-3 | 0.3 | 37.3 | 18.70 | 11.26 | 28.13 |
| SD-4 | 0.4 | 33.9 | 17.10 | 11.44 | 28.06 |
| SD-2 | 0.2 | 25.7 | 14.40 | 8.89 | 22.12 |
| GL-6 | 0.6 | 42.8 | 20.19 | 14.24 | 34.59 |
| GL-5 | 0.5 | 37.4 | 18.52 | 12.60 | 30.81 |
| GL-4 | 0.4 | 35.3 | 16.93 | 11.11 | 27.35 |
| GL-3 | 0.3 | 30.8 | 15.53 | 9.83 | 24.35 |
| GL-2 | 0.2 | 26.1 | 14.34 | 8.77 | 21.85 |
| GR-6 | 0.6 | 36.1 | 18.40 | 13.51 | 32.56 |
| GR-5 | 0.5 | 32.5 | 17.18 | 12.07 | 29.34 |
| GR-4 | 0.4 | 29.7 | 15.98 | 10.75 | 26.34 |
| GR-3 | 0.3 | 27.4 | 14.90 | 9.61 | 23.73 |
| GR-2 | 0.2 | 23.0 | 13.97 | 8.65 | 21.51 |
| LI-6 | 0.6 | 23.4 | 12.99 | 9.71 | 23.32 |
| LI-5 | 0.5 | 23.9 | 12.91 | 9.22 | 22.34 |
| LI-4 | 0.4 | 22.3 | 12.78 | 8.72 | 21.31 |
| LI-3 | 0.3 | 20.6 | 12.66 | 8.26 | 20.35 |
| LI-2 | 0.2 | 20.2 | 12.57 | 7.85 | 19.49 |
| PB-5 | 0.5 | 16.2 | 21.39 | 6.17 | 16.89 |
| PB-4 | 0.4 | 16.7 | 18.89 | 6.34 | 17.11 |
| PB-2 | 0.2 | 17.2 | 15.06 | 6.72 | 17.55 |

TABLE 3

Effect of Volume Fraction of the Transition Zone

| $v_2$ | $k_v/E_3$ | $\mu_v/E_3$ | $E_v/E_3$ | $k_r/E_3$ | $\mu_r/E_3$ | $E_r/E_3$ | $k_4/E_3$ | $\mu_4/E_3$ | $E_4/E_3$ |
|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 2.502 | 1.155 | 3.002 | 1.390 | 0.642 | 1.668 | 1.579 | 0.799 | 2.051 |
| 0.10 | 2.461 | 1.136 | 2.953 | 1.194 | 0.551 | 1.433 | 1.369 | 0.700 | 1.794 |
| 0.20 | 2.419 | 1.116 | 2.902 | 1.042 | 0.481 | 1.251 | 1.214 | 0.632 | 1.616 |
| 0.30 | 2.377 | 1.097 | 2.853 | 0.928 | 0.428 | 1.113 | 1.096 | 0.578 | 1.475 |
| 0.40 | 2.336 | 1.078 | 2.803 | 0.835 | 0.386 | 1.002 | 0.998 | 0.533 | 1.357 |
| 0.50 | 2.294 | 1.059 | 2.753 | 0.758 | 0.350 | 0.910 | 0.913 | 0.492 | 1.251 |

transition zone (assuming a constant stiffness) on the overall elastic moduli of concrete. Thus, in the following parametric study, the transition zone volume fraction ($v_2$) is varied from 0 to 50%. The aggregate volume fraction ($v_1$) is assumed to be 50% and Poisson's ratio for all phases is taken as 0.3. The ratio of Young's moduli of the aggregate and matrix $(E_1/E_3)$ is taken as 5.0 and that of transition zone and matrix $(E_2/E_3)$ is taken as 0.5.

In Table 3 $k_v$, $\mu_v$, $E_v$ denote bulk, shear and Young's moduli predicted by the Voigt model and $k_r$, $\mu_r$, $E_r$ denote those corresponding to the Reuss model respectively. The first row of Table 3 shows the ratio of elastic moduli of the composite model to that of the matrix $(E_4/E_3)$, in which the transition zone volume fraction is negligible. As it can be seen from the above table, the increase in transition zone volume fraction by 50% reduces the Young's moduli of the four phase composite model by 40%. It can also be seen from the above table that the elastic moduli lie within those predicted by the Voigt and Reuss models.

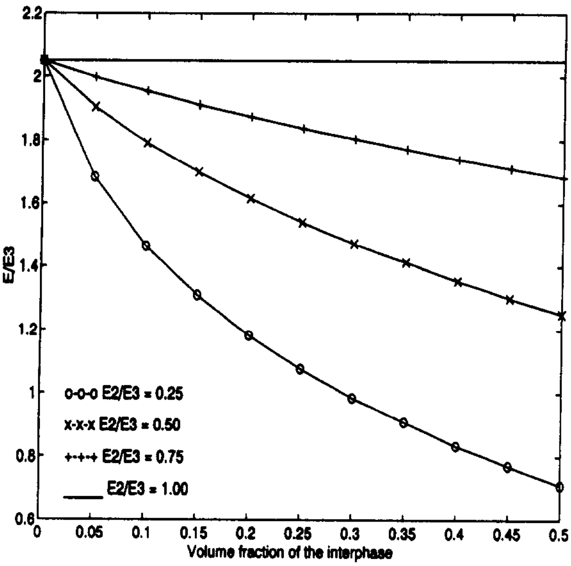


FIG. 3.
Effect of volume fraction and stiffness of the transition zone

TABLE 4

Properties of PC Mortar

| | |
|---|---|
| Volume content of aggregate (sand) | 37% |
| Volume content of transition zone | 10% |
| Young's modulus of sand ($E_1$) | 14.0 e06 psi |
| Young's modulus of PC paste ($E_3$) | 2.7 e06 psi |
| Young's modulus of PC mortar ($E_4$) | 4.2 e06 psi |
| Poisson's ratio of aggregate ($\nu_1$) | 0.1 |
| Poisson's ratio of paste ($\nu_3$) | 0.2 |
| Poisson's ratio of transition zone ($\nu_2$) | 0.2 |
| Poisson's ratio of PC mortar ($\nu_4$) | 0.2 |

Effect of Increase of Stiffness and Volume Fraction of Transition Zone on Elastic Moduli of Concrete. The objective of this study is to simulate the effect of the increase of transition zone stiffness and its volume on the overall elastic moduli of mortar or concrete. Thus, in this parametric study, the ratio of Young's modulus of the transition zone to that of matrix ($E_2/E_3$) is varied from 0.0 to 1.0. The same assumptions about the aggregate volume fraction, and the ratios of Young's moduli of transition zone and aggregate to that of the matrix as in the preceding example are made. Fig. 3 shows the results. It can be seen from this figure that the rate of change of Young's modulus of concrete decreases with increase in the volume fraction of the transition zone. Also, for any given value of volume fraction of the transition zone, for an increase in $E_2/E_3$ ratio, the rate of change of Young's modulus decreases. A similar kind of experimental observation is seen from the plots in the work of Cohen et al. [22].

Evaluation of Transition Zone Stiffness. In practice, the thickness of the transition zone can be measured by using the Scanning Electron Microscopy (SEM) technique. At present, no instrumentation is available that can measure the elastic moduli of the transition zone [22]. However, knowing the overall elastic moduli of concrete or mortar, the composite constitutive model should enable one to determine the elastic moduli of the transition zone.

Cohen et al. [2] have tested portland cement (PC) mortar and silica fume (SF) mortar specimens and evaluated numerically the Young's modulus of the transition zone. In the present

TABLE 5

Properties of SF Mortar

| | |
|---|---|
| Volume content of aggregate (sand) | 37% |
| Volume content of transition zone | 10% |
| Young's modulus of sand ($E_1$) | 14.0 e06 psi |
| Young's modulus of SF paste ($E_3$) | 2.5 e06 psi |
| Young's modulus of SF mortar ($E_4$) | 4.4 e06 psi |
| Poisson's ratio of aggregate ($\nu_1$) | 0.1 |
| Poisson's ratio of paste ($\nu_3$) | 0.2 |
| Poisson's ratio of transition zone ($\nu_2$) | 0.2 |
| Poisson's ratio of SF mortar ($\nu_4$) | 0.2 |

TABLE 6

Elastic Moduli of PC Mortar by Four Phase Composite Model

| $E_2$ | $k_2$ $\times 10^6$ psi | $\mu_2$ $\times 10^6$ psi | $k_4$ $\times 10^6$ psi | $\mu_4$ $\times 10^6$ psi | $E_4$ $\times 10^6$ psi |
|---|---|---|---|---|---|
| 0.1 | 0.150 | 0.113 | 1.269 | 1.028 | 2.428 |
| 0.2 | 0.300 | 0.225 | 1.626 | 1.319 | 3.115 |
| 0.3 | 0.450 | 0.338 | 1.839 | 1.497 | 3.532 |
| 0.4 | 0.600 | 0.450 | 1.980 | 1.619 | 3.817 |
| 0.5 | 0.750 | 0.563 | 2.082 | 1.709 | 4.026 |
| 0.6 | 0.900 | 0.675 | 2.158 | 1.777 | 4.183 |
| 0.7 | 1.050 | 0.788 | 2.218 | 1.831 | 4.308 |
| 0.8 | 1.200 | 0.900 | 2.266 | 1.876 | 4.411 |
| 0.9 | 1.350 | 1.013 | 2.306 | 1.913 | 4.500 |
| 1.0 | 1.500 | 1.125 | 2.340 | 1.944 | 4.567 |

work, the elastic moduli of these specimens are estimated from the four phase composite model using the data given in the Tables 4 and 5. Computing the shear modulus of the transition zone in the four phase composite model knowing the shear modulus of the overall composite, involves a polynomial equation involving fourth power in the shear modulus of the transition zone. Hence, an alternative approach is used in this study. For various values of the ratio $E_2/E_3$, the Young's moduli of the resulting mortar can be tabulated. Thus, for a given value of Young's modulus of the mortar, the corresponding shear modulus of the transition zone can be calculated by linear interpolation. Tables 6 and 7 give the results of elastic moduli of PC and SF mortars respectively for various ratios of $E_2/E_3$ as predicted by the four phase composite model. For the given values of Young's moduli of PC and SF mortars, from Tables

TABLE 7

Elastic Moduli of SF Mortar by Four Phase Composite Model

| $E_2$ | $k_2$ $\times 10^6$ psi | $\mu_2$ $\times 10^6$ psi | $k_4$ $\times 10^6$ psi | $\mu_4$ $\times 10^6$ psi | $E_4$ $\times 10^6$ psi |
|---|---|---|---|---|---|
| 0.1 | 0.139 | 0.104 | 1.182 | 0.956 | 2.259 |
| 0.2 | 0.277 | 0.208 | 1.521 | 1.232 | 2.910 |
| 0.3 | 0.417 | 0.313 | 1.725 | 1.403 | 3.311 |
| 0.4 | 0.556 | 0.417 | 1.861 | 1.518 | 3.580 |
| 0.5 | 0.694 | 0.521 | 1.959 | 1.604 | 3.780 |
| 0.6 | 0.833 | 0.625 | 2.033 | 1.669 | 3.931 |
| 0.7 | 0.972 | 0.729 | 2.091 | 1.721 | 4.051 |
| 0.8 | 1.111 | 0.833 | 2.137 | 1.764 | 4.150 |
| 0.9 | 1.250 | 0.938 | 2.176 | 1.800 | 4.233 |
| 1.0 | 1.389 | 1.042 | 2.208 | 1.830 | 4.302 |
| 1.1 | 1.528 | 1.146 | 2.236 | 1.856 | 4.361 |
| 1.2 | 1.667 | 1.250 | 2.260 | 1.878 | 4.412 |
| 1.3 | 1.806 | 1.354 | 2.281 | 1.898 | 4.458 |

6 and 7, the matching ratios of $E_2/E_3$ for PC and SF mortar cases are found to be 0.61 and 1.18 respectively. Thus the Young's modulus of the transition zone in PC and SF mortar cases is found to be 1.66e06 psi and 2.95e06 psi respectively. This result from the four phase composite model indicates that the addition of silica fume enhances the Young's modulus of the transition zone in SF mortar more than that of the transition zone in PC mortar. Increase in Young's modulus of the transition zone leads to overall increase in Young's modulus of the mortar.

## Summary and Conclusions

The proposed four phase composite model can be used for estimation of elastic moduli of concrete or mortar consisting of three different phases of aggregate, matrix and transition zone. Alternatively, as shown in this paper, one can evaluate the elastic moduli of transition zone knowing those of concrete or mortar. However, more experimental work is needed for estimation of elastic moduli of the transition zone in order to rigorously validate this model. It is possible to introduce the effect of porosity in this constitutive model using the principles of damage mechanics.

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