Pergamon

Cement and Concrete Research, Vol. 26, No. 11, pp. 1629-1637, 1996

0008-8846/96 $15.00 + .00

PII S0008-8846(96)00167-6

FACTORS DETERMINING THE CORRELATIONS BETWEEN CONCRETE PROPERTIES

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(Refereed)
(Received July 23, 1996; in final form September 17, 1996)

ABSTRACT

The factors determining the reliability of the correlations between concrete properties have been investigated. It is assumed that the existence of some resemblance between these properties may affect the strength of the correlations and it is found that the closeness of the sensitivity degrees of the properties to the pore structure of cement paste is the most important factor. Taking into consideration the closeness of dependences of the properties on compositional factors as well as the closeness of their sensitivity degrees, it has been found that it is possible to explain the reliability of the correlations between them with a great degree of confidence.

Introduction

Concrete is a complex material. Its mechanical properties depend upon its pore structure and the nature of the bonds between its components (1,2). As a result, load carrying capacity of such a composite material depends greatly upon the properties of participating constituents and also on the interaction between them. Mechanical behavior, especially strength and fracture of concrete generally exhibits the most important and complex dependency on the structure of the hardened cement paste (3,4). However, the structure of cement paste and the mechanical properties of concrete have generally been studied separetely in the past (5). Recent research efforts on the relations between mechanical behavior and the structure of concrete have provided new ways of understanding material behavior. Modern approaches based on concepts of material science can be used if experiments, theories and numerical studies are combined efficiently (3). There has been a substantial advancement in understanding of mechanical properties especially strength and fracture, in the past few years (1,3,5-10) and it is expected that further advances will be made through a better understanding of the cementitious materials: how the microstructure affects the properties and how the materials, constituents and fabrication techniques can alter microstructure (6-8,11-13).

Relations between concrete properties are often used in engineering applications. Generally, from a readily measurable property, another property is predicted by a regression analysis as it is done in nondestructive methods. For such applications, it is very important that coefficients of correlation between measured and predicted properties are significant. In this paper the factors determining correlation coefficients between concrete properties are investigated. It has been thought that there should be some close resemblances between properties related with high correlations and such resemblances have been systematically investigated.

TABLE 1

Relations Between Properties and Their Coefficients of Correlation

Pairs of Properties	Relation	Coefficient of correlation
V_c-Δ	$\log V_c = \log(1.64\times10^{-6}) + 1.92 \log\Delta$	0.834
V_s-Δ	$\log V_s = \log(1.61\times10^{-6}) + 1.92 \log\Delta$	0.834
S-Δ	$\log S = \log(8.75\times10^{-7}) + 2.25 \log\Delta$	0.570
f_{cc}-Δ	$\log f_{cc} = \log(1.04\times10^{-18}) + 5.77 \log\Delta$	0.684
f_{cs}-Δ	$\log f_{cs} = \log(1.17\times10^{-18}) + 5.74 \log\Delta$	0.702
T_s-Δ	$\log T_s = \log(2.89\times10^{-15}) + 4.43 \log\Delta$	0.656
E-Δ	$\log E = \log(8.12\times10^{-12}) + 4.63 \log\Delta$	0.768
V_s-V_c	$\log V_s = \log 1.04 + 0.962 \log V_c$	0.959
S-V_c	$\log S = \log 5.21 + 1.18 \log V_c$	0.686
f_{cc}-V_c	$\log f_{cc} = \log 0.213 + 3.12 \log V_c$	0.850
f_{cs}-V_c	$\log f_{cs} = \log 0.215 + 3.04 \log V_c$	0.853
T_s-V_c	$\log T_s = \log 0.0617 + 2.34 \log V_c$	0.795
E-V_c	$\log E = \log 776 + 2.37 \log V_c$	0.901
S-V_s	$\log S = \log 5.46 + 1.16 \log V_s$	0.677
f_{cc}-V_s	$\log f_{cc} = \log 0.275 + 2.98 \log V_s$	0.814
f_{cs}-V_s	$\log f_{cs} = \log 0.254 + 2.96 \log V_s$	0.833
T_s-V_s	$\log T_s = \log 0.0740 + 2.24 \log V_s$	0.765
E-V_s	$\log E = \log 674 + 2.49 \log V_s$	0.950
f_{cc}-S	$\log f_{cc} = \log 0.0586 + 1.74 \log S$	0.815
f_{cs}-S	$\log f_{cs} = \log 0.0719 + 1.65 \log S$	0.796
T_s-S	$\log T_s = \log 0.0200 + 1.35 \log S$	0.791
E-S	$\log E = \log 911 + 0.989 \log S$	0.646
f_{cs}-f_{cc}	$\log f_{cs} = \log 1.076 + 0.939 \log f_{cc}$	0.969
T_s-f_{cc}	$\log T_s = \log 0.197 + 0.748 \log f_{cc}$	0.935
E-f_{cc}	$\log E = \log 5020 + 0.535 \log f_{cc}$	0.748
T_s-f_{cs}	$\log T_s = \log 0.202 + 0.770 \log f_{cs}$	0.932
E-f_{cs}	$\log E = \log 4840 + 0.568 \log f_{cs}$	0.770
E-T_s	$\log E = \log 17200 + 0.611 \log T_s$	0.684

Relations Between Properties

Relations and Coefficients of Correlation Between Properties. Eight properties of 60 mixtures given in Table 1 of Ref. (14) have been paired and thus 28 relations between them are established. For this, the following relation has been set for any two P_{hi} and P_{hj} properties:

$$P_{hi} = A\left(P_{hj}\right)^{B} \tag{1}$$

Here A and B are constants. The relation given in Eq.(1) has been changed to

$$\log P_{hi} = \log A + B\log P_{hj} \tag{2}$$

and r_{ij} coefficients of correlation of this linear logarithmic form have been calculated. The results obtained can altogether be seen in Table 1.

The following properties have been measured from each mixture on the 28th day: i) Unit weight (Δ), ii) Ultrasonic pulse velocity perpendicular to the casting direction measured on cubes (V_c), iii) Ultrasonic pulse velocity on cylinders measured in the direction of the axis (V_s), iv) Rebound number by Schmidt's Hammer (S) measured on cubes, v) Splitting tensile strength measured on cylinders (T_s), vi) Cube compressive strength (f_{cc}), vii) Cylinder compressive strength (f_{cs}), viii) Modulus of elasticity (E) under compressive loading measured on cylindrical specimens. Unit weight is given in kg/m^3, ultrasonic pulse velocities are in km/sec, the strengths and the modulus of elasticity in N/mm^2.

TABLE 2

(t_{ij}) Coefficients of Sensitivity Closeness According to $(SD2)_i$'s

Pairs of Properties	According to func.				Pairs of Properties	According to func.			
	(3)	(4)	(5)	(6)		(3)	(4)	(5)	(6)
V_c-Δ	0.928	0.928	0.938	0.946	f_{cc}-V_s	0.951	0.929	0.890	0.917
V_s-Δ	0.963	0.962	0.967	0.972	f_{cs}-V_s	0.987	0.964	0.932	0.954
S-Δ	0.930	0.883	0.832	0.874	T_s-V_s	0.988	0.980	0.952	0.971
f_{cc}-Δ	0.916	0.894	0.860	0.891	E-V_s	0.972	0.971	0.972	0.975
f_{cs}-Δ	0.950	0.928	0.901	0.927	f_{cc}-S	0.985	0.989	0.967	0.981
T_s-Δ	0.952	0.943	0.920	0.944	f_{cs}-S	0.978	0.952	0.923	0.943
E-Δ	0.936	0.934	0.940	0.948	T_s-S	0.977	0.937	0.904	0.926
V_s-V_c	0.963	0.965	0.970	0.972	E-S	0.993	0.946	0.886	0.922
S-V_c	0.998	0.952	0.888	0.924	f_{cs}-f_{cc}	0.964	0.963	0.954	0.961
f_{cc}-V_c	0.987	0.963	0.918	0.943	T_s-f_{cc}	0.962	0.948	0.935	0.944
f_{cs}-V_c	0.976	1.000	0.961	0.981	E-f_{cc}	0.978	0.957	0.916	0.940
T_s-V_c	0.975	0.984	0.982	0.998	T_s-f_{cs}	0.999	0.984	0.979	0.983
E-V_c	0.991	0.993	0.998	0.998	E-f_{cs}	0.985	0.993	0.959	0.978
S-V_s	0.965	0.918	0.861	0.899	E-T_s	0.984	0.991	0.979	0.996

The Effect of the Closeness of the Sensitivity Degrees of Properties on the Strength of the Correlations

Coefficient of Sensitivity Closeness. Type 1 sensitivity degree $(SD1)_i$ of an P_{hi} concrete property to the structure of cement paste and Type 2 sensitivity degree $(SD2)_i$ were defined in Ref. (14). Let $(SD)_i$ be a more general symbol representing Type 1 or Type 2 sensitivity degree of any P_{hi} concrete property. Let us call t_{ij}, defined by $t_{ij} = (SD)_i / (SD)_j$ and $t_{ij} \leq 1$, the coefficient of sensitivity closeness of P_{hi} and P_{hi} properties on the pore structure of cement paste. According to the above definition, t_{ij} coefficients take values between 0 and 1. A t_{ij} value close to 1 shows that P_{hi} and P_{hj} properties are close to each other from the viewpoint of the sensitivity to the structure of cement paste.

In Ref. (14) the dependence of concrete properties on factors related to internal structure have been analyzed according to the following equations:

$$P_{hi} = A(kw + a) + B \tag{3}$$

$$P_{hi} = A\left(\frac{C}{kw + a}\right) + B \tag{4}$$

$$P_{hi} = A(c + w + a) + B\left(\frac{c}{kw + a}\right) + C \tag{5}$$

$$P_{hi} = A(c + w + a) + B\left(\frac{c}{kw + a}\right) + Cm + D \tag{6}$$

where w, a and c are the volumes of water, air and cement in 1 m^3 compacted concrete respectively and A,B,C and D are constants. The above functions were applied to the test results of 60 different mixtures, and $(SD2)_i$ values were calculated and given in Ref. (14). Here by using those values, the closeness coefficients t_{ij} are calculated for each function and given in Table 2.

As stated in Ref. (14), the $(SD1)_i$'s do not have stable values with the different functions. Furthermore analysis presented in this paper showed that they are less successful than $(SD2)_i$'s in clarifying the correlations between properties. For these reasons only the $(SD2)_i$'s are considered in Table 2 and in following calculations.

Correlations Between (t_{ij})s and (r_{ij})s. If the closeness of sensitivities of concrete properties to the pore structure of cement paste has an effect on the reliability of correlations between them, then there should be good correlations between r_{ij} coefficients and t_{ij} coefficients. Accordingly linear regression between r_{ij} values given in Table 1 and t_{ij} values given in Table 2 have been analysed by using the least squares method and the correlation coefficients and regression relations between (r_{ij})s and (t_{ij})s obtained are given in Table 3.

These have been calculated between 28 pairs of values. For the coefficient of correlation for 28 observations to be significant at the 5% level it should be not less than 0.374 and at the 1% level it should be not less than 0.478 (15). Therefore the values obtained for Eqs. (4), (5) and (6) are found to be significant at the 1% level ($R>0.478$). Among these, the highest one is the value of 0.684 obtained for Eq. (5). Although there are more variables in Eq. (6), the coefficient of correlation obtained is lower.

TABLE 3

Coefficients of Correlation and Linear Relations Between (r_{ij})s and (t_{ij})s Obtained Using $(SD2)_i$'s.

Equation	Relation	Coefficient of correlation
Eq. (3)	t_{ij}=0.930 + 0.049 r_{ij}	0.233
Eq. (4)	t_{ij}=0.840 + 0.146 r_{ij}	0.515
Eq. (5)	t_{ij}=0.714 + 0.274 r_{ij}	0.684
Eq. (6)	t_{ij}=0.806 + 0.181 r_{ij}	0.589

When the variables in the functions are revised the situation existing may be interpreted thus: The closeness of $(SD2)_i$s explains the correlations between concrete properties as much as the number of paste-related variables increases in functions. When other variables are added, related to the structure but not directly to the cement paste (Such as the fineness modulus in Eq. (6)), the success of $(SD2)_i$'s decreases.

In Ref. (14), it was stated that $(SD2)_i$ sensitivity degrees obtained from various functions were a little different from each other; and to find which was more significant, it would be necessary to compare the coefficients of correlation between properties with their closenesses of their sensitivity degrees. As seen here, the most reliable relation with r_{ij}'s can be set with t_{ij} coefficients obtained through the Eq. (5). Hence, it can be concluded that the most significant sensitivity degrees can be obtained from the equations including as many variables as possible for the cement paste phase and including only these.

The Effect of the Closeness of Dependences on Composition Factors of Properties on the Strength of the Correlations

Above it is observed that there are reliable relations between the closeness of the sensitivity degrees of concrete properties on the pore structure of cement paste and the coefficients of correlations between these properties. Do relations between these coefficients of correlation and the resemblances of properties from other aspects exist? To find out the answer, the significance, for concrete properties, of changes in composition factors considered in this work have been calculated; and relations between the closeness of influences of these changes on different properties and the coefficients of correlation between them have been analysed.

Let us consider the equation

$$(P_{hi})' = A(c+w+a)' + B\left(\frac{c}{kw+a}\right)' + C(m)' + D \tag{7}$$

Let variables with (′) show the ratio of the value of the variable in parenthesis in any mixture to the average of that variable in all of the 60 mixtures. In this case, coefficients A,B and C define how the property P_{hi} is affected by changes in volume of cement paste, the pore structure of cement paste and grading of aggregate respectively.

Using the results obtained from the 60 concrete mixtures considered here the relation above has been used for each property. Relations and coefficients of correlations are given altogether in Table 4. Coefficients A,B and C given in Table 4 can be thought as the "slopes" of linear relations considered. To decide how close these coefficients are for any two properties, it has been thought appropriate to calculate the absolute values of the differences of these "slopes". Thus for any two different i and j properties, the following values have been calculated:

$$\begin{aligned} T(A)_{ij} &= | A_i - A_j | \\ T(B)_{ij} &= | B_i - B_j | \\ T(C)_{ij} &= | C_i - C_j | \end{aligned} \qquad (8)$$

When T values are high, coefficients A,B and C considered for I and j properties are different. In this case, in the variation of the concerned composition factors, the properties i and j are thought to be influenced in a different way. In Table 5, eight properties are paired and the $T(A)_{ij}$, $T(B)_{ij}$ and $T(C)_{ij}$ values obtained are given. These are then paired with r_{ij} coefficients of correlation given in Table 1 and thus linear correlations between the following variables have been calculated.

$$\begin{aligned} &T(A)_{ij} - r_{ij} \\ &T(B)_{ij} - r_{ij} \\ &T(C)_{ij} - r_{ij} \end{aligned} \qquad (9)$$

TABLE 4

$(P_{hi})' = A(c+w+a)' + B\left(\frac{c}{kw+a}\right)' + C(m)' + D$ Relations and Coefficients of Correlation

Property	Relation	Coefficient of correlation
Δ	$(\Delta)'=-0.0483(c+w+a)'+0.0743(c/0.45w+a)'+0.0876(m)'+0.887$	0.908
V_c	$(V_c)'=-0.0853(c+w+a)'+0.289(c/2.75w+a)'+0.185(m)'+0.612$	0.908
V_s	$(V_s)'=-0.0892(c+w+a)'+0.252(c/1.50w+a)'+0.173(m)'+0.664$	0.865
S	$(S)'=-0.0946(c+w+a)'+0.656(c/1.65w+a)'-0.0669(m)'+0.507$	0.775
f_{cc}	$(f_{cc})'=-0.215(c+w+a)'+1.54(c/1.90w+a)'+0.0371(m)'-0.359$	0.882
f_{cs}	$(f_{cs})'=-0.267(c+w+a)'+1.38(c/1.45w+a)'+0.0741(m)'-0.190$	0.834
T_s	$(T_s)'=-0.244(c+w+a)'+1.03(c/1.25w+a)'+0.0217(m)'+0.195$	0.801
E	$(E)'=-0.165(c+w+a)'+0.665(c/1.45w+a)'+0.421(m)'+0.0782$	0.807

TABLE 5

$T(A)_{ij}$, $T(B)_{ij}$ and $T(C)_{ij}$ Values of Pairs of Properties

Pairs of properties	$T(A)_{ij}$	$T(B)_{ij}$	$T(C)_{ij}$
Δ-V_c	0.037	0.215	0.097
Δ-V_s	0.041	0.178	0.085
Δ-S	0.046	0.581	0.155
Δ-f_{cc}	0.167	1.461	0.051
Δ-f_{cs}	0.219	1.310	0.013
Δ-T_s	0.195	0.953	0.066
Δ-E	0.117	0.591	0.333
V_c-V_s	0.004	0.037	0.012
V_c-S	0.009	0.366	0.252
V_c-f_{cc}	0.130	1.247	0.148
V_c-f_{cs}	0.182	1.095	0.111
V_c-T_s	0.158	0.738	0.163
V_c-E	0.080	0.376	0.236
V_s-S	0.005	0.404	0.240
V_s-f_{cc}	0.126	1.284	0.136
V_s-f_{cs}	0.178	1.132	0.098
V_s-T_s	0.154	0.775	0.151
V_s-E	0.076	0.413	0.248
S-f_{cc}	0.120	0.880	0.104
S-f_{cs}	0.173	0.728	0.141
S-T_s	0.149	0.372	0.089
S-E	0.071	0.010	0.488
f_{cc}-f_{cs}	0.052	0.152	0.037
f_{cc}-T_s	0.029	0.508	0.015
f_{cc}-E	0.050	0.870	0.384
f_{cs}-T_s	0.024	0.356	0.052
f_{cs}-E	0.102	0.718	0.347
T_s-E	0.078	0.362	0.399

TABLE 6

$T(A)_{ij}$-r_{ij}, $T(B)_{ij}$-r_{ij}, $T(C)_{ij}$-r_{ij} Relations and Correlation Coefficients

Variables	Relation	Coefficient of correlation
$T(A)_{ij}$-r_{ij}	$T(A)_{ij}$=0.196-0.122 r_{ij}	0.200
$T(B)_{ij}$-r_{ij}	$T(B)_{ij}$=1.34-0.867 r_{ij}	0.223
$T(C)_{ij}$-r_{ij}	$T(C)_{ij}$=0.553-0.488 r_{ij}	0.400

Relations and coefficients of correlation obtained can be seen in Table 6.

The first two coefficients of correlation are not significant at the 5% level and that obtained for $T(C)_{ij}$-r_{ij} is significant at between the 5% and 1% levels.

So, it can be said that there is a weak relation between the closenness of the influence of aggregate mixture fineness modulus (or the grading of aggregate mixture) on any two properties and coefficients of correlation between these two properties. For other factors of mixture, not even such a weak relation can be found. In this case the following question rises: What is the situation when the closenesses of influences from changes of composition factors are considered all together? In order to understand this, the (r_{ij})s have been related to $T(A)_{ij}$, $T(B)_{ij}$ and $T(C)_{ij}$ by means of linear multiple regression analysis. The relation obtained is

$$r_{ij} = -0.17124\ T(A)_{ij} - 0.06638\ T(B)_{ij} - 0.39476\ T(C)_{ij} + 0.918930$$

The correlation coefficient is 0.521. This coefficient is slightly above the value of 0.478 required for significance at the 1% level. Nevertheless, it is lower substantially than the greatest value of 0.684 which was obtained considering the closenesses of the properties from the aspect of their sensitivity to the structure of cement paste.

How can the correlations between properties be clarified by considering closenesses from both the aspect of sensitivity and of influencing from changes in composition factors all together? In order to understand this, the (r_{ij})s have been related to (t_{ij})'s and to $T(A)_{ij}$, $T(B)_{ij}$ and $T(C)_{ij}$s by means of linear multiple regression analysis where (t_{ij})s have been obtained from the values of $(SD2)_i$'s using the Eq. (5). In this case the relation obtained is

$$r_{ij} = -0.61823\ T(A)ij + 0.039311\ T(B)_{ij} - 0.34144\ T(C)_{ij} + 1.754116\ t_{ij} - 0.74812$$

The coefficient of correlation is 0.83 which is considerably higher than that necessary for significance at the 1% level.

Conclusions

The results obtained in this work can be summarized as follows:

1- $(SD2)_i$ type sensitivity degrees are more succesful in clarifying the correlations between properties, than $(SD1)_i$ type 1 sensivity degrees.

2- Among the functions relating concrete properties to internal structure factors, providing the calculation of the most significant sensivity degrees are the ones which include the most variables for the cement paste phase only.
3- The most important reason for the strength of the correlations between concrete properties is the closeness of their sensitivity to the pore structure of cement paste.

The closenesses of the properties from the aspect of influencing from the variations in composition factors are not sufficiently successful in clarifying the correlations when these factors are considered individually. Nevertheless when the factors are considered all together, quite good correlation is obtained even though it is not quite as significant as that involving sensitivity to the structure of cement paste.

Taking into consideration the closeness of the dependences of properties on composition factors as well as the closeness of their sensitivity to the pore structure of cement paste, it is possible to explain the strongness or the weakness of the correlations between them with a great degree of confidence.

Acknowledgement

The authors would like to thank Professor Folker H.Wittmann for his great help in the first draft of this paper. They also wish to express their thanks to Mr. Frank D. Lydon for editing and comments introduced into the paper and acknowledge the assistances of Professor M.Hulusi Özkul, Mr. Hakan Gülseren and Mr. Yildiray Süvari.

References

1. S.P. Skalny (Editor), Materials Science of Concrete I, Am. Ceram. Soc., (1989).
2. J.W. Dougill, in Struct. Mech. in Reactor Technol., F.H. Wittmann, ed., A.A.Balkema, Rotterdam, 59 (1987).
3. F.H. Wittmann, Mater. and Struct., 20, 163 (1987).
4. F.H. Wittmann (Editor), Fracture Mechanics of Concrete, Elsevier, Amsterdam, (1983).
5. T.C. Hansen, Mater. and Struct., 19, 423 (1986).
6. H.M. Jennings and D.A. Lange, in New Horizons in Constr. Mater., Proc., ASCE National Convention, St.Louis, MO, 4(1988).
7. H.M. Jennings, Mater. Sci. and Tech., 4, 285 (1988).
8. S. Sarkar, in Advances in Concrete Tech., V.M.Malhotra, ed., CANMET, 125 (1994).
9. P.K. Metha, Concrete-Structure, Properties and Materials, Prentice-Hall Inc., New Jersey, (1986).
10. S. Diamond, in Proc. of 8th Int. Cong. on Chem. of Cement, Vol.1, Rio de Janeiro, 122 (1986).
11. M. Regourd, in Mater.Res.Sym.Proc. Vol.42, 3(1985).
12. L. Struble, in Mater. Res. Soc., Symp.Proc., S.Mindess and S.P.Shah, eds., 114, 11 (1988).
13. S.P. Shah and J.F. Young, Ceram. Bull., 69, 1319 (1990).
14. O.N. Oktar, H. Moral and M.A. Taşdemir, Sensitivity of Concrete Properties to the Pore Structure of Hardened Cement Paste, Cem. Conc. Res., Accepted for publication (1996).
15. B. Postacĭoglu, Essentials of Building Materials, Istanbul Technical University Publications, in Turkish (1966).